Vogliamo risolvere le equazoni differenziali supponendo che le soluzioni siano analitiche, ossia supponendo che il loro sviluppo di Taylor conincida con la funzione. Per semplicità e per meglio comprendere come funziona il meccanismo, cominciamo con l'equazione differenziale $f'(t) = f(t)$.

In [1]:
```
f=function('f')(x)
```

In [2]:
```
f.series(x,5)
```

Out[2]:
```
(f(0)) + (D[0](f)(0))*x + (1/2*D[0, 0](f)(0))*x^2 + (1/6*D[0, 0, 0](f)(0))*x^3 + (1/2
4*D[0, 0, 0, 0](f)(0))*x^4 + Order(x^5)
```

In [3]:
```
q(x)=f.series(x,5)
q(x)
```

Out[3]:
```
1/24*x^4*D[0, 0, 0, 0](f)(0) + 1/6*x^3*D[0, 0, 0](f)(0) + 1/2*x^2*D[0, 0](f)(0) + x*D
[0](f)(0) + f(0)
```

In [4]:
```
expand(q(x)^2).coefficients()
```

Out[4]:
```
[[f(0)^2, 0],
 [2*f(0)*D[0](f)(0), 1],
 [D[0](f)(0)^2 + f(0)*D[0, 0](f)(0), 2],
 [D[0](f)(0)*D[0, 0](f)(0) + 1/3*f(0)*D[0, 0, 0](f)(0), 3],
 [1/4*D[0, 0](f)(0)^2 + 1/3*D[0](f)(0)*D[0, 0, 0](f)(0) + 1/12*f(0)*D[0, 0, 0,
0](f)(0),
  4],
 [1/6*D[0, 0](f)(0)*D[0, 0, 0](f)(0) + 1/12*D[0](f)(0)*D[0, 0, 0, 0](f)(0), 5],
 [1/36*D[0, 0, 0](f)(0)^2 + 1/24*D[0, 0](f)(0)*D[0, 0, 0, 0](f)(0), 6],
 [1/72*D[0, 0, 0](f)(0)*D[0, 0, 0, 0](f)(0), 7],
 [1/576*D[0, 0, 0, 0](f)(0)^2, 8]]
```

In [5]:
```
(f^2).series(x,5)
```

Out[5]:
```
(f(0)^2) + (2*f(0)*D[0](f)(0))*x + (D[0](f)(0)^2 + f(0)*D[0, 0](f)(0))*x^2 + (D[0](f)
(0)*D[0, 0](f)(0) + 1/3*f(0)*D[0, 0, 0](f)(0))*x^3 + (1/4*D[0, 0](f)(0)^2 + 1/3*D
[0](f)(0)*D[0, 0, 0](f)(0) + 1/12*f(0)*D[0, 0, 0, 0](f)(0))*x^4 + Order(x^5)
```

In [6]:
```
(sin(f).series(x,5))
```

Out[6]:
```
(sin(f(0))) + (cos(f(0))*D[0](f)(0))*x + (-1/2*sin(f(0))*D[0](f)(0)^2 + 1/2*cos(f(0))*
D[0, 0](f)(0))*x^2 + (-1/6*cos(f(0))*D[0](f)(0)^3 - 1/2*sin(f(0))*D[0](f)(0)*D[0,
0](f)(0) + 1/6*cos(f(0))*D[0, 0, 0](f)(0))*x^3 + (1/24*sin(f(0))*D[0](f)(0)^4 - 1/4*co
s(f(0))*D[0](f)(0)^2*D[0, 0](f)(0) - 1/8*sin(f(0))*D[0, 0](f)(0)^2 - 1/6*sin(f(0))*D
[0](f)(0)*D[0, 0, 0](f)(0) + 1/24*cos(f(0))*D[0, 0, 0, 0](f)(0))*x^4 + Order(x^5)
```

In [7]:
```
(diff(f(x),x)-f(x)).series(x,5)
```

Out[7]:
```
(-f(0) + D[0](f)(0)) + (-D[0](f)(0) + D[0, 0](f)(0))*x + (-1/2*D[0, 0](f)(0) + 1/2*D
[0, 0, 0](f)(0))*x^2 + (-1/6*D[0, 0, 0](f)(0) + 1/6*D[0, 0, 0, 0](f)(0))*x^3 + (-1/24*
D[0, 0, 0, 0](f)(0) + 1/24*D[0, 0, 0, 0, 0](f)(0))*x^4 + Order(x^5)
```

In [8]:
```
(diff(f(x),x)-f(x)).series(x,5).coefficients()
```

Out[8]:
```
[[-f(0) + D[0](f)(0), 0],
 [-D[0](f)(0) + D[0, 0](f)(0), 1],
 [-1/2*D[0, 0](f)(0) + 1/2*D[0, 0, 0](f)(0), 2],
 [-1/6*D[0, 0, 0](f)(0) + 1/6*D[0, 0, 0, 0](f)(0), 3],
 [-1/24*D[0, 0, 0, 0](f)(0) + 1/24*D[0, 0, 0, 0, 0](f)(0), 4]]
```

Ora dovremmo risolvere le precedenti equazioni, dove le incognite sono le derivate di f in 0.

Pertanto se vogliamo risolvere la $f'(t) = f(t)^2$ per serie, scriviamo

In [9]:
```
(diff(f,x)-f^2).series(x,5).coefficients()
```

Out[9]:
```
[[-f(0)^2 + D[0](f)(0), 0],
 [-2*f(0)*D[0](f)(0) + D[0, 0](f)(0), 1],
 [-D[0](f)(0)^2 - f(0)*D[0, 0](f)(0) + 1/2*D[0, 0, 0](f)(0), 2],
 [-D[0](f)(0)*D[0, 0](f)(0) - 1/3*f(0)*D[0, 0, 0](f)(0) + 1/6*D[0, 0, 0, 0](f)(0),
  3],
 [-1/4*D[0, 0](f)(0)^2 - 1/3*D[0](f)(0)*D[0, 0, 0](f)(0) - 1/12*f(0)*D[0, 0, 0,
0](f)(0) + 1/24*D[0, 0, 0, 0, 0](f)(0),
  4]]
```

In [72]:
```

```

Out[72]:

Useremo ora un approccio un po' diverso. Usando le serie di potenze formali.

Cominciamo, sempre per semplicità, con $f' = f$

In [10]:
```
a=[var('a'+str(j)) for j in range(40)]
```

In [11]:
```
R.<t> = SR[[]]
```

In [12]:
```
f=R(a,40)
f
```

Out[12]:
```
a0 + a1*t + a2*t^2 + a3*t^3 + a4*t^4 + a5*t^5 + a6*t^6 + a7*t^7 + a8*t^8 + a9*t^9 + a1
0*t^10 + a11*t^11 + a12*t^12 + a13*t^13 + a14*t^14 + a15*t^15 + a16*t^16 + a17*t^17 +
a18*t^18 + a19*t^19 + a20*t^20 + a21*t^21 + a22*t^22 + a23*t^23 + a24*t^24 + a25*t^25
+ a26*t^26 + a27*t^27 + a28*t^28 + a29*t^29 + a30*t^30 + a31*t^31 + a32*t^32 + a33*t^3
3 + a34*t^34 + a35*t^35 + a36*t^36 + a37*t^37 + a38*t^38 + a39*t^39 + O(t^40)
```

In [13]:
```
pol=f.derivative(t)-f
pol
```

Out[13]:
```
-a0 + a1 + (-a1 + 2*a2)*t + (-a2 + 3*a3)*t^2 + (-a3 + 4*a4)*t^3 + (-a4 + 5*a5)*t^4 +
(-a5 + 6*a6)*t^5 + (-a6 + 7*a7)*t^6 + (-a7 + 8*a8)*t^7 + (-a8 + 9*a9)*t^8 + (10*a10 -
a9)*t^9 + (-a10 + 11*a11)*t^10 + (-a11 + 12*a12)*t^11 + (-a12 + 13*a13)*t^12 + (-a13 +
14*a14)*t^13 + (-a14 + 15*a15)*t^14 + (-a15 + 16*a16)*t^15 + (-a16 + 17*a17)*t^16 + (-
a17 + 18*a18)*t^17 + (-a18 + 19*a19)*t^18 + (-a19 + 20*a20)*t^19 + (-a20 + 21*a21)*t^2
0 + (-a21 + 22*a22)*t^21 + (-a22 + 23*a23)*t^22 + (-a23 + 24*a24)*t^23 + (-a24 + 25*a2
5)*t^24 + (-a25 + 26*a26)*t^25 + (-a26 + 27*a27)*t^26 + (-a27 + 28*a28)*t^27 + (-a28 +
29*a29)*t^28 + (-a29 + 30*a30)*t^29 + (-a30 + 31*a31)*t^30 + (-a31 + 32*a32)*t^31 + (-
a32 + 33*a33)*t^32 + (-a33 + 34*a34)*t^33 + (-a34 + 35*a35)*t^34 + (-a35 + 36*a36)*t^3
5 + (-a36 + 37*a37)*t^36 + (-a37 + 38*a38)*t^37 + (-a38 + 39*a39)*t^38 + O(t^39)
```

In [14]:
```
cc=(pol.coefficients())
cc
```

Out[14]:
```
[-a0 + a1,
 -a1 + 2*a2,
 -a2 + 3*a3,
 -a3 + 4*a4,
 -a4 + 5*a5,
 -a5 + 6*a6,
 -a6 + 7*a7,
 -a7 + 8*a8,
 -a8 + 9*a9,
 10*a10 - a9,
 -a10 + 11*a11,
 -a11 + 12*a12,
 -a12 + 13*a13,
 -a13 + 14*a14,
 -a14 + 15*a15,
 -a15 + 16*a16,
 -a16 + 17*a17,
 -a17 + 18*a18,
 -a18 + 19*a19,
 -a19 + 20*a20,
 -a20 + 21*a21,
 -a21 + 22*a22,
 -a22 + 23*a23,
 -a23 + 24*a24,
 -a24 + 25*a25,
 -a25 + 26*a26,
 -a26 + 27*a27,
 -a27 + 28*a28,
 -a28 + 29*a29,
 -a29 + 30*a30,
 -a30 + 31*a31,
 -a31 + 32*a32,
 -a32 + 33*a33,
 -a33 + 34*a34,
 -a34 + 35*a35,
 -a35 + 36*a36,
 -a36 + 37*a37,
 -a37 + 38*a38,
 -a38 + 39*a39]
```

Imponiamo la condizione iniziale $f(0) = 1$

In [15]:
```
cc.extend([a0-1])
solve(cc,a)
```

Out[15]:
```
[[a0 == 1, a1 == 1, a2 == (1/2), a3 == (1/6), a4 == (1/24), a5 == (1/120), a6 == (1/72
0), a7 == (1/5040), a8 == (1/40320), a9 == (1/362880), a10 == (1/3628800), a11 == (1/3
9916800), a12 == (1/479001600), a13 == (1/6227020800), a14 == (1/87178291200), a15 ==
(1/1307674368000), a16 == (1/20922789888000), a17 == (1/355687428096000), a18 == (1/64
02373705728000), a19 == (1/121645100408832000), a20 == (1/2432902008176640000), a21 ==
(1/51090942171709440000), a22 == (1/1124000727777607680000), a23 == (1/258520167388849
76640000), a24 == (1/620448401733239439360000), a25 == (1/15511210043330985984000000),
a26 == (1/403291461126605635584000000), a27 == (1/10888869450418352160768000000), a28
== (1/304888344611713860501504000000), a29 == (1/8841761993739701954543616000000), a30
== (1/265252859812191058636308480000000), a31 == (1/822283865417792281772556288000000
0), a32 == (1/263130836933693530167218012160000000), a33 == (1/868331761881188649551819
4401280000000), a34 == (1/295232799039604140847618609643520000000), a35 == (1/1033314
79663861449296666513375232000000000), a36 == (1/371993326789901217467999448150835200000
000), a37 == (1/13763753091226345046315979581580902400000000), a38 == (1/5230226174666
0111176000722410007429120000000), a39 == (1/20397882081197443358640281739902897356800
000000)]]
```

In [79]:
```

```

Out[79]:

Cerchiamo di risolvere $f'' = -f$

In [31]:
```
pol=f.derivative(t,t)+f
pol
```

Out[31]:
```
a0 + 2*a2 + (a1 + 6*a3)*t + (a2 + 12*a4)*t^2 + (a3 + 20*a5)*t^3 + (a4 + 30*a6)*t^4 +
(a5 + 42*a7)*t^5 + (a6 + 56*a8)*t^6 + (a7 + 72*a9)*t^7 + (90*a10 + a8)*t^8 + (110*a11
+ a9)*t^9 + (a10 + 132*a12)*t^10 + (a11 + 156*a13)*t^11 + (a12 + 182*a14)*t^12 + (a13
+ 210*a15)*t^13 + (a14 + 240*a16)*t^14 + (a15 + 272*a17)*t^15 + (a16 + 306*a18)*t^16 +
(a17 + 342*a19)*t^17 + (a18 + 380*a20)*t^18 + (a19 + 420*a21)*t^19 + (a20 + 462*a22)*t
^20 + (a21 + 506*a23)*t^21 + (a22 + 552*a24)*t^22 + (a23 + 600*a25)*t^23 + (a24 + 650*
a26)*t^24 + (a25 + 702*a27)*t^25 + (a26 + 756*a28)*t^26 + (a27 + 812*a29)*t^27 + (a28
+ 870*a30)*t^28 + (a29 + 930*a31)*t^29 + (a30 + 992*a32)*t^30 + (a31 + 1056*a33)*t^31
+ (a32 + 1122*a34)*t^32 + (a33 + 1190*a35)*t^33 + (a34 + 1260*a36)*t^34 + (a35 + 1332*
a37)*t^35 + (a36 + 1406*a38)*t^36 + (a37 + 1482*a39)*t^37 + O(t^38)
```

In [3]:
```
cc=(pol.coefficients())
cc
```

Out[3]:
```
[a0 + 2*a2,
 a1 + 6*a3,
 a2 + 12*a4,
 a3 + 20*a5,
 a4 + 30*a6,
 a5 + 42*a7,
 a6 + 56*a8,
 a7 + 72*a9,
 90*a10 + a8,
 110*a11 + a9,
 a10 + 132*a12,
 a11 + 156*a13,
 a12 + 182*a14,
 a13 + 210*a15,
 a14 + 240*a16,
 a15 + 272*a17,
 a16 + 306*a18,
 a17 + 342*a19,
 a18 + 380*a20,
 a19 + 420*a21,
 a20 + 462*a22,
 a21 + 506*a23,
 a22 + 552*a24,
 a23 + 600*a25,
 a24 + 650*a26,
 a25 + 702*a27,
 a26 + 756*a28,
 a27 + 812*a29,
 a28 + 870*a30,
 a29 + 930*a31,
 a30 + 992*a32,
 a31 + 1056*a33,
 a32 + 1122*a34,
 a33 + 1190*a35,
 a34 + 1260*a36,
 a35 + 1332*a37,
 a36 + 1406*a38,
 a37 + 1482*a39]
```

Cerchiamo la soluzione con le condizioni iniziali $f(0) = 1$ ed $f'(0) = 0$

In [83]:
```
solve(cc+[a0-1,a1-0],a)
```

Out[83]:
```
[[a0 == 1, a1 == 0, a2 == (-1/2), a3 == 0, a4 == (1/24), a5 == 0, a6 == (-1/720), a7
== 0, a8 == (1/40320), a9 == 0, a10 == (-1/3628800), a11 == 0, a12 == (1/479001600), a
13 == 0, a14 == (-1/87178291200), a15 == 0, a16 == (1/20922789888000), a17 == 0, a18
== (-1/6402373705728000), a19 == 0, a20 == (1/2432902008176640000), a21 == 0, a22 ==
(-1/1124000727777607680000), a23 == 0, a24 == (1/620448401733239439360000), a25 == 0,
a26 == (-1/403291461126605635584000000), a27 == 0, a28 == (1/304888344611713860501504000000), a29 == 0, a30 == (-1/265252859812191058636308480000000), a31 == 0, a32 == (1/2
63130836933693530167218012160000000), a33 == 0, a34 == (-1/2952327990396041408476186096
43520000000), a35 == 0, a36 == (1/371993326789901217467999448150835200000000), a37 ==
0, a38 == (-1/523022617466601111760007224100074291200000000), a39 == 0]]
```

Cerchiamo la soluzione con le condizioni iniziali $f(0) = 0$ ed $f'(0) = 1$

In [84]:
```
solve(cc+[a0-0,a1-1],a)
```

Out[84]:
```
[[a0 == 0, a1 == 1, a2 == 0, a3 == (-1/6), a4 == 0, a5 == (1/120), a6 == 0, a7 == (-1/
5040), a8 == 0, a9 == (1/362880), a10 == 0, a11 == (-1/39916800), a12 == 0, a13 == (1/
6227020800), a14 == 0, a15 == (-1/1307674368000), a16 == 0, a17 == (1/355687428096000
0), a18 == 0, a19 == (-1/121645100408832000), a20 == 0, a21 == (1/51090942171709440000
0), a22 == 0, a23 == (-1/25852016738884976640000), a24 == 0, a25 == (1/155112100433309
85984000000), a26 == 0, a27 == (-1/10888869450418352160768000000), a28 == 0, a29 == (1
/8841761993739701954543616000000), a30 == 0, a31 == (-1/82228386541779228177255628800
0000), a32 == 0, a33 == (1/868331761881188649551819440128000000), a34 == 0, a35 == (-
1/10333147966386144929666651337523200000000), a36 == 0, a37 == (1/13763753091226345046
315979581580902400000000), a38 == 0, a39 == (-1/20397882081197443358640281739902897356
800000000)]]
```

Risolviamo $f'' = -\lambda^2 f$ con dati iniziali $f(0) = 1$ e $f'(0) = 0$

In [36]:
```
l=var('l')
pol=f.derivative(t,t)+l^2*f
pol
```

Out[36]:
```
a0*l^2 + 2*a2 + (a1*l^2 + 6*a3)*t + (a2*l^2 + 12*a4)*t^2 + (a3*l^2 + 20*a5)*t^3 + (a4*
l^2 + 30*a6)*t^4 + (a5*l^2 + 42*a7)*t^5 + (a6*l^2 + 56*a8)*t^6 + (a7*l^2 + 72*a9)*t^7
+ (a8*l^2 + 90*a10)*t^8 + (a9*l^2 + 110*a11)*t^9 + (a10*l^2 + 132*a12)*t^10 + (a11*l^2
+ 156*a13)*t^11 + (a12*l^2 + 182*a14)*t^12 + (a13*l^2 + 210*a15)*t^13 + (a14*l^2 + 24
0*a16)*t^14 + (a15*l^2 + 272*a17)*t^15 + (a16*l^2 + 306*a18)*t^16 + (a17*l^2 + 342*a1
9)*t^17 + (a18*l^2 + 380*a20)*t^18 + (a19*l^2 + 420*a21)*t^19 + (a20*l^2 + 462*a22)*t^
20 + (a21*l^2 + 506*a23)*t^21 + (a22*l^2 + 552*a24)*t^22 + (a23*l^2 + 600*a25)*t^23 +
(a24*l^2 + 650*a26)*t^24 + (a25*l^2 + 702*a27)*t^25 + (a26*l^2 + 756*a28)*t^26 + (a27*
l^2 + 812*a29)*t^27 + (a28*l^2 + 870*a30)*t^28 + (a29*l^2 + 930*a31)*t^29 + (a30*l^2 +
992*a32)*t^30 + (a31*l^2 + 1056*a33)*t^31 + (a32*l^2 + 1122*a34)*t^32 + (a33*l^2 + 119
0*a35)*t^33 + (a34*l^2 + 1260*a36)*t^34 + (a35*l^2 + 1332*a37)*t^35 + (a36*l^2 + 1406*
a38)*t^36 + (a37*l^2 + 1482*a39)*t^37 + O(t^38)
```

In [27]:
```
cc=(pol.coefficients())
cc
```

Out[27]:
```
[a0*l^2 + 2*a2,
 a1*l^2 + 6*a3,
 a2*l^2 + 12*a4,
 a3*l^2 + 20*a5,
 a4*l^2 + 30*a6,
 a5*l^2 + 42*a7,
 a6*l^2 + 56*a8,
 a7*l^2 + 72*a9,
 a8*l^2 + 90*a10,
 a9*l^2 + 110*a11,
 a10*l^2 + 132*a12,
 a11*l^2 + 156*a13,
 a12*l^2 + 182*a14,
 a13*l^2 + 210*a15,
 a14*l^2 + 240*a16,
 a15*l^2 + 272*a17,
 a16*l^2 + 306*a18,
 a17*l^2 + 342*a19,
 a18*l^2 + 380*a20,
 a19*l^2 + 420*a21,
 a20*l^2 + 462*a22,
 a21*l^2 + 506*a23,
 a22*l^2 + 552*a24,
 a23*l^2 + 600*a25,
 a24*l^2 + 650*a26,
 a25*l^2 + 702*a27,
 a26*l^2 + 756*a28,
 a27*l^2 + 812*a29,
 a28*l^2 + 870*a30,
 a29*l^2 + 930*a31,
 a30*l^2 + 992*a32,
 a31*l^2 + 1056*a33,
 a32*l^2 + 1122*a34,
 a33*l^2 + 1190*a35,
 a34*l^2 + 1260*a36,
 a35*l^2 + 1332*a37,
 a36*l^2 + 1406*a38,
 a37*l^2 + 1482*a39]
```

In [86]:
```
solve(cc+[a0-1,a1-0],a)
```

Out[86]:
```
[[a0 == 1, a1 == 0, a2 == -1/2*l^2, a3 == 0, a4 == 1/24*l^4, a5 == 0, a6 == -1/720*l^
6, a7 == 0, a8 == 1/40320*l^8, a9 == 0, a10 == -1/3628800*l^10, a11 == 0, a12 == 1/479
001600*l^12, a13 == 0, a14 == -1/87178291200*l^14, a15 == 0, a16 == 1/20922789888000*l
^16, a17 == 0, a18 == -1/6402373705728000*l^18, a19 == 0, a20 == 1/243290200817664000
0*l^20, a21 == 0, a22 == -1/1124000727777607680000*l^22, a23 == 0, a24 == 1/6204484017
33239439360000*l^24, a25 == 0, a26 == -1/403291461126605635584000000*l^26, a27 == 0, a
28 == 1/304888344611713860501504000000*l^28, a29 == 0, a30 == -1/265252859812191058636
308480000000*l^30, a31 == 0, a32 == 1/263130836933693530167218012160000000*l^32, a33
== 0, a34 == -1/295232799039604140847618609643520000000*l^34, a35 == 0, a36 == 1/37199
332678990121746799944815083520000000000*l^36, a37 == 0, a38 == -1/5230226174666011117600
07224100074291200000000*l^38, a39 == 0]]
```

In [119]:
```
```

Out[119]:

Vogliamo studiare l'equazioni non omogenea $f'' = -f + \sin(2t)$

In [16]:
```
cs=(sin(2*x).series(x,40)).coefficients()
```

In [17]:
```
csin=[x[0] for x in cs]
```

In [18]:
```
ssin=R(csin,40)
```

In [116]:
```
ssin
```

Out[116]:
```
2*t - 4/3*t^3 + 4/15*t^5 - 8/315*t^7 + 4/2835*t^9 - 8/155925*t^11 + 8/6081075*t^13 - 1
6/638512875*t^15 + 4/10854718875*t^17 - 8/1856156927625*t^19 + 8/194896477400625*t^21
- 16/49308808782358125*t^23 + 8/3698160658676859375*t^25 - 16/129805439119557640625*t
^27 + 16/2635050414127022610046875*t^29 - 32/122529844256906551386796875*t^31 + 4/40434
8486047791619576429 6875*t^33 - 8/2405873491984360136479756640625*t^35 + 8/801155872830
79192544775896132 8125*t^37 - 16/59365650176761681675678939034414 0625*t^39 + O(t^40)
```

In [19]:
```
pol=f.derivative(t,t)+f-ssin
cc=pol.coefficients()
solve(cc+[a0-0,a1-0],a)
```

Out[19]:
```
[[a0 == 0, a1 == 0, a2 == 0, a3 == (1/3), a4 == 0, a5 == (-1/12), a6 == 0, a7 == (1/12
0), a8 == 0, a9 == (-17/36288), a10 == 0, a11 == (31/1814400), a12 == 0, a13 == (-1/22
80960), a14 == 0, a15 == (5461/653837184000), a16 == 0, a17 == (-257/2092278988800), a
18 == 0, a19 == (73/50812489728000), a20 == 0, a21 == (-1271/92892622130380800), a22
== 0, a23 == (60787/56200036388880384 0000), a24 == 0, a25 == (-241/3342212894490624000
00), a26 == 0, a27 == (22369621/544443472520917608038400000 0), a28 == 0, a29 == (-6170
93/304888344611713860501504000 00), a30 == 0, a31 == (49981/574140389203876750295040000
000), a32 == 0, a33 == (-16843009/510783389341875676206952611840 00000), a34 == 0, a35
== (5726623061/516657398319307246483332566876160 0000000), a36 == 0, a37 == (-7957/2390
54898007776632265278226432 0000000), a38 == 0, a39 == (91625968981/1019894104059872167 9
3201408699514486784 00000000)]]
```

Studiamo l'equazione nonlineare $f' = f^2$

```
In [20]: pol=f.derivative(t)-f^2
         pol
```

```
Out[20]: -a0^2 + a1 + (-2*a0*a1 + 2*a2)*t + (-a1^2 - 2*a0*a2 + 3*a3)*t^2 + (-2*a1*a2 - 2*a0*a3
+ 4*a4)*t^3 + (-a2^2 - 2*a1*a3 - 2*a0*a4 + 5*a5)*t^4 + (-2*a2*a3 - 2*a1*a4 - 2*a0*a5 +
6*a6)*t^5 + (-a3^2 - 2*a2*a4 - 2*a1*a5 - 2*a0*a6 + 7*a7)*t^6 + (-2*a3*a4 - 2*a2*a5 -
2*a1*a6 - 2*a0*a7 + 8*a8)*t^7 + (-a4^2 - 2*a3*a5 - 2*a2*a6 - 2*a1*a7 - 2*a0*a8 + 9*a
9)*t^8 + (-2*a4*a5 - 2*a3*a6 - 2*a2*a7 - 2*a1*a8 - 2*a0*a9 + 10*a10)*t^9 + (-2*a0*a10
- a5^2 - 2*a4*a6 - 2*a3*a7 - 2*a2*a8 - 2*a1*a9 + 11*a11)*t^10 + (-2*a1*a10 - 2*a0*a11
- 2*a5*a6 - 2*a4*a7 - 2*a3*a8 - 2*a2*a9 + 12*a12)*t^11 + (-2*a1*a11 - 2*a0*a12 - 2*a1
0*a2 - a6^2 - 2*a5*a7 - 2*a4*a8 - 2*a3*a9 + 13*a13)*t^12 + (-2*a1*a12 - 2*a0*a13 - 2*a
11*a2 - 2*a10*a3 - 2*a6*a7 - 2*a5*a8 - 2*a4*a9 + 14*a14)*t^13 + (-2*a1*a13 - 2*a0*a14
- 2*a12*a2 - 2*a11*a3 - 2*a10*a4 - a7^2 - 2*a6*a8 - 2*a5*a9 + 15*a15)*t^14 + (-2*a1*a1
4 - 2*a0*a15 - 2*a13*a2 - 2*a12*a3 - 2*a11*a4 - 2*a10*a5 - 2*a7*a8 - 2*a6*a9 + 16*a1
6)*t^15 + (-2*a1*a15 - 2*a0*a16 - 2*a14*a2 - 2*a13*a3 - 2*a12*a4 - 2*a11*a5 - 2*a10*a6
- a8^2 - 2*a7*a9 + 17*a17)*t^16 + (-2*a1*a16 - 2*a0*a17 - 2*a15*a2 - 2*a14*a3 - 2*a13*
a4 - 2*a12*a5 - 2*a11*a6 - 2*a10*a7 - 2*a8*a9 + 18*a18)*t^17 + (-2*a1*a17 - 2*a0*a18 -
2*a16*a2 - 2*a15*a3 - 2*a14*a4 - 2*a13*a5 - 2*a12*a6 - 2*a11*a7 - 2*a10*a8 - a9^2 + 1
9*a19)*t^18 + (-2*a1*a18 - 2*a0*a19 - 2*a17*a2 - 2*a16*a3 - 2*a15*a4 - 2*a14*a5 - 2*a1
3*a6 - 2*a12*a7 - 2*a11*a8 - 2*a10*a9 + 20*a20)*t^19 + (-a10^2 - 2*a1*a19 - 2*a18*a2 -
2*a0*a20 - 2*a17*a3 - 2*a16*a4 - 2*a15*a5 - 2*a14*a6 - 2*a13*a7 - 2*a12*a8 - 2*a11*a9
+ 21*a21)*t^20 + (-2*a10*a11 - 2*a19*a2 - 2*a1*a20 - 2*a0*a21 - 2*a18*a3 - 2*a17*a4 -
2*a16*a5 - 2*a15*a6 - 2*a14*a7 - 2*a13*a8 - 2*a12*a9 + 22*a22)*t^21 + (-a11^2 - 2*a10*
a12 - 2*a2*a20 - 2*a1*a21 - 2*a0*a22 - 2*a19*a3 - 2*a18*a4 - 2*a17*a5 - 2*a16*a6 - 2*a
15*a7 - 2*a14*a8 - 2*a13*a9 + 23*a23)*t^22 + (-2*a11*a12 - 2*a10*a13 - 2*a2*a21 - 2*a
1*a22 - 2*a0*a23 - 2*a20*a3 - 2*a19*a4 - 2*a18*a5 - 2*a17*a6 - 2*a16*a7 - 2*a15*a8 -
2*a14*a9 + 24*a24)*t^23 + (-a12^2 - 2*a11*a13 - 2*a10*a14 - 2*a2*a22 - 2*a1*a23 - 2*a
0*a24 - 2*a21*a3 - 2*a20*a4 - 2*a19*a5 - 2*a18*a6 - 2*a17*a7 - 2*a16*a8 - 2*a15*a9 + 2
5*a25)*t^24 + (-2*a12*a13 - 2*a11*a14 - 2*a10*a15 - 2*a2*a23 - 2*a1*a24 - 2*a0*a25 -
2*a22*a3 - 2*a21*a4 - 2*a20*a5 - 2*a19*a6 - 2*a18*a7 - 2*a17*a8 - 2*a16*a9 + 26*a26)*t
^25 + (-a13^2 - 2*a12*a14 - 2*a11*a15 - 2*a10*a16 - 2*a2*a24 - 2*a1*a25 - 2*a0*a26 -
2*a23*a3 - 2*a22*a4 - 2*a21*a5 - 2*a20*a6 - 2*a19*a7 - 2*a18*a8 - 2*a17*a9 + 27*a27)*t
^26 + (-2*a13*a14 - 2*a12*a15 - 2*a11*a16 - 2*a10*a17 - 2*a2*a25 - 2*a1*a26 - 2*a0*a27
- 2*a24*a3 - 2*a23*a4 - 2*a22*a5 - 2*a21*a6 - 2*a20*a7 - 2*a19*a8 - 2*a18*a9 + 28*a2
8)*t^27 + (-a14^2 - 2*a13*a15 - 2*a12*a16 - 2*a11*a17 - 2*a10*a18 - 2*a2*a26 - 2*a1*a2
7 - 2*a0*a28 - 2*a25*a3 - 2*a24*a4 - 2*a23*a5 - 2*a22*a6 - 2*a21*a7 - 2*a20*a8 - 2*a1
9*a9 + 29*a29)*t^28 + (-2*a14*a15 - 2*a13*a16 - 2*a12*a17 - 2*a11*a18 - 2*a10*a19 - 2*
a2*a27 - 2*a1*a28 - 2*a0*a29 - 2*a26*a3 - 2*a25*a4 - 2*a24*a5 - 2*a23*a6 - 2*a22*a7 -
2*a21*a8 - 2*a20*a9 + 30*a30)*t^29 + (-a15^2 - 2*a14*a16 - 2*a13*a17 - 2*a12*a18 - 2*a
11*a19 - 2*a10*a20 - 2*a2*a28 - 2*a1*a29 - 2*a27*a3 - 2*a0*a30 - 2*a26*a4 - 2*a25*a5 -
2*a24*a6 - 2*a23*a7 - 2*a22*a8 - 2*a21*a9 + 31*a31)*t^30 + (-2*a15*a16 - 2*a14*a17 -
2*a13*a18 - 2*a12*a19 - 2*a11*a20 - 2*a10*a21 - 2*a2*a29 - 2*a28*a3 - 2*a1*a30 - 2*a0*
a31 - 2*a27*a4 - 2*a26*a5 - 2*a25*a6 - 2*a24*a7 - 2*a23*a8 - 2*a22*a9 + 32*a32)*t^31 +
(-a16^2 - 2*a15*a17 - 2*a14*a18 - 2*a13*a19 - 2*a12*a20 - 2*a11*a21 - 2*a10*a22 - 2*a2
9*a3 - 2*a2*a30 - 2*a1*a31 - 2*a0*a32 - 2*a28*a4 - 2*a27*a5 - 2*a26*a6 - 2*a25*a7 - 2*
a24*a8 - 2*a23*a9 + 33*a33)*t^32 + (-2*a16*a17 - 2*a15*a18 - 2*a14*a19 - 2*a13*a20 -
2*a12*a21 - 2*a11*a22 - 2*a10*a23 - 2*a3*a30 - 2*a2*a31 - 2*a1*a32 - 2*a0*a33 - 2*a29*
a4 - 2*a28*a5 - 2*a27*a6 - 2*a26*a7 - 2*a25*a8 - 2*a24*a9 + 34*a34)*t^33 + (-a17^2 -
2*a16*a18 - 2*a15*a19 - 2*a14*a20 - 2*a13*a21 - 2*a12*a22 - 2*a11*a23 - 2*a10*a24 - 2*
a3*a31 - 2*a2*a32 - 2*a1*a33 - 2*a0*a34 - 2*a30*a4 - 2*a29*a5 - 2*a28*a6 - 2*a27*a7 -
2*a26*a8 - 2*a25*a9 + 35*a35)*t^34 + (-2*a17*a18 - 2*a16*a19 - 2*a15*a20 - 2*a14*a21 -
2*a13*a22 - 2*a12*a23 - 2*a11*a24 - 2*a10*a25 - 2*a3*a32 - 2*a2*a33 - 2*a1*a34 - 2*a0*
a35 - 2*a31*a4 - 2*a30*a5 - 2*a29*a6 - 2*a28*a7 - 2*a27*a8 - 2*a26*a9 + 36*a36)*t^35 +
(-a18^2 - 2*a17*a19 - 2*a16*a20 - 2*a15*a21 - 2*a14*a22 - 2*a13*a23 - 2*a12*a24 - 2*a1
1*a25 - 2*a10*a26 - 2*a3*a33 - 2*a2*a34 - 2*a1*a35 - 2*a0*a36 - 2*a32*a4 - 2*a31*a5 -
2*a30*a6 - 2*a29*a7 - 2*a28*a8 - 2*a27*a9 + 37*a37)*t^36 + (-2*a18*a19 - 2*a17*a20 -
2*a16*a21 - 2*a15*a22 - 2*a14*a23 - 2*a13*a24 - 2*a12*a25 - 2*a11*a26 - 2*a10*a27 - 2*
a3*a34 - 2*a2*a35 - 2*a1*a36 - 2*a0*a37 - 2*a33*a4 - 2*a32*a5 - 2*a31*a6 - 2*a30*a7 -
2*a29*a8 - 2*a28*a9 + 38*a38)*t^37 + (-a19^2 - 2*a18*a20 - 2*a17*a21 - 2*a16*a22 - 2*a
15*a23 - 2*a14*a24 - 2*a13*a25 - 2*a12*a26 - 2*a11*a27 - 2*a10*a28 - 2*a3*a35 - 2*a2*a
36 - 2*a1*a37 - 2*a0*a38 - 2*a34*a4 - 2*a33*a5 - 2*a32*a6 - 2*a31*a7 - 2*a30*a8 - 2*a2
9*a9 + 39*a39)*t^38 + O(t^39)
```

In [89]:

```
cc=(pol.coefficients())
cc
```

```
Out[89]: [-a0^2 + a1,
 -2*a0*a1 + 2*a2,
 -a1^2 - 2*a0*a2 + 3*a3,
 -2*a1*a2 - 2*a0*a3 + 4*a4,
 -a2^2 - 2*a1*a3 - 2*a0*a4 + 5*a5,
 -2*a2*a3 - 2*a1*a4 - 2*a0*a5 + 6*a6,
 -a3^2 - 2*a2*a4 - 2*a1*a5 - 2*a0*a6 + 7*a7,
 -2*a3*a4 - 2*a2*a5 - 2*a1*a6 - 2*a0*a7 + 8*a8,
 -a4^2 - 2*a3*a5 - 2*a2*a6 - 2*a1*a7 - 2*a0*a8 + 9*a9,
 -2*a4*a5 - 2*a3*a6 - 2*a2*a7 - 2*a1*a8 - 2*a0*a9 + 10*a10,
 -2*a0*a10 - a5^2 - 2*a4*a6 - 2*a3*a7 - 2*a2*a8 - 2*a1*a9 + 11*a11,
 -2*a1*a10 - 2*a0*a11 - 2*a5*a6 - 2*a4*a7 - 2*a3*a8 - 2*a2*a9 + 12*a12,
 -2*a1*a11 - 2*a0*a12 - 2*a10*a2 - a6^2 - 2*a5*a7 - 2*a4*a8 - 2*a3*a9 + 13*a13,
 -2*a1*a12 - 2*a0*a13 - 2*a11*a2 - 2*a10*a3 - 2*a6*a7 - 2*a5*a8 - 2*a4*a9 + 14*a14,
 -2*a1*a13 - 2*a0*a14 - 2*a12*a2 - 2*a11*a3 - 2*a10*a4 - a7^2 - 2*a6*a8 - 2*a5*a9 + 1
5*a15,
 -2*a1*a14 - 2*a0*a15 - 2*a13*a2 - 2*a12*a3 - 2*a11*a4 - 2*a10*a5 - 2*a7*a8 - 2*a6*a9
+ 16*a16,
 -2*a1*a15 - 2*a0*a16 - 2*a14*a2 - 2*a13*a3 - 2*a12*a4 - 2*a11*a5 - 2*a10*a6 - a8^2 -
2*a7*a9 + 17*a17,
 -2*a1*a16 - 2*a0*a17 - 2*a15*a2 - 2*a14*a3 - 2*a13*a4 - 2*a12*a5 - 2*a11*a6 - 2*a10*a
7 - 2*a8*a9 + 18*a18,
 -2*a1*a17 - 2*a0*a18 - 2*a16*a2 - 2*a15*a3 - 2*a14*a4 - 2*a13*a5 - 2*a12*a6 - 2*a11*a
7 - 2*a10*a8 - a9^2 + 19*a19,
 -2*a1*a18 - 2*a0*a19 - 2*a17*a2 - 2*a16*a3 - 2*a15*a4 - 2*a14*a5 - 2*a13*a6 - 2*a12*a
7 - 2*a11*a8 - 2*a10*a9 + 20*a20,
 -a10^2 - 2*a1*a19 - 2*a18*a2 - 2*a0*a20 - 2*a17*a3 - 2*a16*a4 - 2*a15*a5 - 2*a14*a6 -
2*a13*a7 - 2*a12*a8 - 2*a11*a9 + 21*a21,
 -2*a10*a11 - 2*a19*a2 - 2*a1*a20 - 2*a0*a21 - 2*a18*a3 - 2*a17*a4 - 2*a16*a5 - 2*a15*
a6 - 2*a14*a7 - 2*a13*a8 - 2*a12*a9 + 22*a22,
 -a11^2 - 2*a10*a12 - 2*a2*a20 - 2*a1*a21 - 2*a0*a22 - 2*a19*a3 - 2*a18*a4 - 2*a17*a5
- 2*a16*a6 - 2*a15*a7 - 2*a14*a8 - 2*a13*a9 + 23*a23,
 -2*a11*a12 - 2*a10*a13 - 2*a2*a21 - 2*a1*a22 - 2*a0*a23 - 2*a20*a3 - 2*a19*a4 - 2*a1
8*a5 - 2*a17*a6 - 2*a16*a7 - 2*a15*a8 - 2*a14*a9 + 24*a24,
 -a12^2 - 2*a11*a13 - 2*a10*a14 - 2*a2*a22 - 2*a1*a23 - 2*a0*a24 - 2*a21*a3 - 2*a20*a4
- 2*a19*a5 - 2*a18*a6 - 2*a17*a7 - 2*a16*a8 - 2*a15*a9 + 25*a25,
 -2*a12*a13 - 2*a11*a14 - 2*a10*a15 - 2*a2*a23 - 2*a1*a24 - 2*a0*a25 - 2*a22*a3 - 2*a2
1*a4 - 2*a20*a5 - 2*a19*a6 - 2*a18*a7 - 2*a17*a8 - 2*a16*a9 + 26*a26,
 -a13^2 - 2*a12*a14 - 2*a11*a15 - 2*a10*a16 - 2*a2*a24 - 2*a1*a25 - 2*a0*a26 - 2*a23*a
3 - 2*a22*a4 - 2*a21*a5 - 2*a20*a6 - 2*a19*a7 - 2*a18*a8 - 2*a17*a9 + 27*a27,
 -2*a13*a14 - 2*a12*a15 - 2*a11*a16 - 2*a10*a17 - 2*a2*a25 - 2*a1*a26 - 2*a0*a27 - 2*a
24*a3 - 2*a23*a4 - 2*a22*a5 - 2*a21*a6 - 2*a20*a7 - 2*a19*a8 - 2*a18*a9 + 28*a28,
 -a14^2 - 2*a13*a15 - 2*a12*a16 - 2*a11*a17 - 2*a10*a18 - 2*a2*a26 - 2*a1*a27 - 2*a0*a
28 - 2*a25*a3 - 2*a24*a4 - 2*a23*a5 - 2*a22*a6 - 2*a21*a7 - 2*a20*a8 - 2*a19*a9 + 29*a
29,
 -2*a14*a15 - 2*a13*a16 - 2*a12*a17 - 2*a11*a18 - 2*a10*a19 - 2*a2*a27 - 2*a1*a28 - 2*
a0*a29 - 2*a26*a3 - 2*a25*a4 - 2*a24*a5 - 2*a23*a6 - 2*a22*a7 - 2*a21*a8 - 2*a20*a9 +
30*a30,
 -a15^2 - 2*a14*a16 - 2*a13*a17 - 2*a12*a18 - 2*a11*a19 - 2*a10*a20 - 2*a2*a28 - 2*a1*
a29 - 2*a27*a3 - 2*a0*a30 - 2*a26*a4 - 2*a25*a5 - 2*a24*a6 - 2*a23*a7 - 2*a22*a8 - 2*a
21*a9 + 31*a31,
 -2*a15*a16 - 2*a14*a17 - 2*a13*a18 - 2*a12*a19 - 2*a11*a20 - 2*a10*a21 - 2*a2*a29 -
2*a28*a3 - 2*a1*a30 - 2*a0*a31 - 2*a27*a4 - 2*a26*a5 - 2*a25*a6 - 2*a24*a7 - 2*a23*a8
- 2*a22*a9 + 32*a32,
 -a16^2 - 2*a15*a17 - 2*a14*a18 - 2*a13*a19 - 2*a12*a20 - 2*a11*a21 - 2*a10*a22 - 2*a2
9*a3 - 2*a2*a30 - 2*a1*a31 - 2*a0*a32 - 2*a28*a4 - 2*a27*a5 - 2*a26*a6 - 2*a25*a7 - 2*
a24*a8 - 2*a23*a9 + 33*a33,
 -2*a16*a17 - 2*a15*a18 - 2*a14*a19 - 2*a13*a20 - 2*a12*a21 - 2*a11*a22 - 2*a10*a23 -
2*a3*a30 - 2*a2*a31 - 2*a1*a32 - 2*a0*a33 - 2*a29*a4 - 2*a28*a5 - 2*a27*a6 - 2*a26*a7
- 2*a25*a8 - 2*a24*a9 + 34*a34,
 -a17^2 - 2*a16*a18 - 2*a15*a19 - 2*a14*a20 - 2*a13*a21 - 2*a12*a22 - 2*a11*a23 - 2*a1
0*a24 - 2*a3*a31 - 2*a2*a32 - 2*a1*a33 - 2*a0*a34 - 2*a30*a4 - 2*a29*a5 - 2*a28*a6 -
2*a27*a7 - 2*a26*a8 - 2*a25*a9 + 35*a35,
 -2*a17*a18 - 2*a16*a19 - 2*a15*a20 - 2*a14*a21 - 2*a13*a22 - 2*a12*a23 - 2*a11*a24 -
2*a10*a25 - 2*a3*a32 - 2*a2*a33 - 2*a1*a34 - 2*a0*a35 - 2*a31*a4 - 2*a30*a5 - 2*a29*a6
- 2*a28*a7 - 2*a27*a8 - 2*a26*a9 + 36*a36,
 -a18^2 - 2*a17*a19 - 2*a16*a20 - 2*a15*a21 - 2*a14*a22 - 2*a13*a23 - 2*a12*a24 - 2*a1
1*a25 - 2*a10*a26 - 2*a3*a33 - 2*a2*a34 - 2*a1*a35 - 2*a0*a36 - 2*a32*a4 - 2*a31*a5 -
2*a30*a6 - 2*a29*a7 - 2*a28*a8 - 2*a27*a9 + 37*a37,
 -2*a18*a19 - 2*a17*a20 - 2*a16*a21 - 2*a15*a22 - 2*a14*a23 - 2*a13*a24 - 2*a12*a25 -
2*a11*a26 - 2*a10*a27 - 2*a3*a34 - 2*a2*a35 - 2*a1*a36 - 2*a0*a37 - 2*a33*a4 - 2*a32*a
5 - 2*a31*a6 - 2*a30*a7 - 2*a29*a8 - 2*a28*a9 + 38*a38,
 -a19^2 - 2*a18*a20 - 2*a17*a21 - 2*a16*a22 - 2*a15*a23 - 2*a14*a24 - 2*a13*a25 - 2*a1
2*a26 - 2*a11*a27 - 2*a10*a28 - 2*a3*a35 - 2*a2*a36 - 2*a1*a37 - 2*a0*a38 - 2*a34*a4 -
2*a33*a5 - 2*a32*a6 - 2*a31*a7 - 2*a30*a8 - 2*a29*a9 + 39*a39]
```

Risolviamo l'equazione richiedendo il dato iniziale $f(0) = 1$

In [88]:
```
solve(cc+[a0-1],a)
```

Out[88]:
```
[[a0 == 1, a1 == 1, a2 == 1, a3 == 1, a4 == 1, a5 == 1, a6 == 1, a7 == 1, a8 == 1, a9
== 1, a10 == 1, a11 == 1, a12 == 1, a13 == 1, a14 == 1, a15 == 1, a16 == 1, a17 == 1,
a18 == 1, a19 == 1, a20 == 1, a21 == 1, a22 == 1, a23 == 1, a24 == 1, a25 == 1, a26 ==
1, a27 == 1, a28 == 1, a29 == 1, a30 == 1, a31 == 1, a32 == 1, a33 == 1, a34 == 1, a35
== 1, a36 == 1, a37 == 1, a38 == 1, a39 == 1]]
```

In [87]:
```
var('k')
%assume(x<1)
sum((x)^k,k,0,oo)
```

Out[87]:
```
-1/(x - 1)
```

Pertanto la soluzione di $f' = f^2$ con $f(0) = 1$ esiste per $0 \leq t < 1$. Ossia la soluzione non esiste per tutti i tempi, ma "esplode" per $t \to 1$

Studiamo la equazione non lineare $f' = f^2 + 1$ e dato iniziale $f(0) = 0$

In [91]:
```
pol=f.derivative(t)-f^2-1
```

Out[91]:

In [102]:

```
cc=(pol.coefficients())
cc
```

```
Out[102]: [-a0^2 + a1 - 1,
 -2*a0*a1 + 2*a2,
 -a1^2 - 2*a0*a2 + 3*a3,
 -2*a1*a2 - 2*a0*a3 + 4*a4,
 -a2^2 - 2*a1*a3 - 2*a0*a4 + 5*a5,
 -2*a2*a3 - 2*a1*a4 - 2*a0*a5 + 6*a6,
 -a3^2 - 2*a2*a4 - 2*a1*a5 - 2*a0*a6 + 7*a7,
 -2*a3*a4 - 2*a2*a5 - 2*a1*a6 - 2*a0*a7 + 8*a8,
 -a4^2 - 2*a3*a5 - 2*a2*a6 - 2*a1*a7 - 2*a0*a8 + 9*a9,
 -2*a4*a5 - 2*a3*a6 - 2*a2*a7 - 2*a1*a8 - 2*a0*a9 + 10*a10,
 -2*a0*a10 - a5^2 - 2*a4*a6 - 2*a3*a7 - 2*a2*a8 - 2*a1*a9 + 11*a11,
 -2*a1*a10 - 2*a0*a11 - 2*a5*a6 - 2*a4*a7 - 2*a3*a8 - 2*a2*a9 + 12*a12,
 -2*a1*a11 - 2*a0*a12 - 2*a10*a2 - a6^2 - 2*a5*a7 - 2*a4*a8 - 2*a3*a9 + 13*a13,
 -2*a1*a12 - 2*a0*a13 - 2*a11*a2 - 2*a10*a3 - 2*a6*a7 - 2*a5*a8 - 2*a4*a9 + 14*a14,
 -2*a1*a13 - 2*a0*a14 - 2*a12*a2 - 2*a11*a3 - 2*a10*a4 - a7^2 - 2*a6*a8 - 2*a5*a9 + 1
5*a15,
 -2*a1*a14 - 2*a0*a15 - 2*a13*a2 - 2*a12*a3 - 2*a11*a4 - 2*a10*a5 - 2*a7*a8 - 2*a6*a9
+ 16*a16,
 -2*a1*a15 - 2*a0*a16 - 2*a14*a2 - 2*a13*a3 - 2*a12*a4 - 2*a11*a5 - 2*a10*a6 - a8^2 -
2*a7*a9 + 17*a17,
 -2*a1*a16 - 2*a0*a17 - 2*a15*a2 - 2*a14*a3 - 2*a13*a4 - 2*a12*a5 - 2*a11*a6 - 2*a10*a
7 - 2*a8*a9 + 18*a18,
 -2*a1*a17 - 2*a0*a18 - 2*a16*a2 - 2*a15*a3 - 2*a14*a4 - 2*a13*a5 - 2*a12*a6 - 2*a11*a
7 - 2*a10*a8 - a9^2 + 19*a19,
 -2*a1*a18 - 2*a0*a19 - 2*a17*a2 - 2*a16*a3 - 2*a15*a4 - 2*a14*a5 - 2*a13*a6 - 2*a12*a
7 - 2*a11*a8 - 2*a10*a9 + 20*a20,
 -a10^2 - 2*a1*a19 - 2*a18*a2 - 2*a0*a20 - 2*a17*a3 - 2*a16*a4 - 2*a15*a5 - 2*a14*a6 -
2*a13*a7 - 2*a12*a8 - 2*a11*a9 + 21*a21,
 -2*a10*a11 - 2*a19*a2 - 2*a1*a20 - 2*a0*a21 - 2*a18*a3 - 2*a17*a4 - 2*a16*a5 - 2*a15*
a6 - 2*a14*a7 - 2*a13*a8 - 2*a12*a9 + 22*a22,
 -a11^2 - 2*a10*a12 - 2*a2*a20 - 2*a1*a21 - 2*a0*a22 - 2*a19*a3 - 2*a18*a4 - 2*a17*a5
- 2*a16*a6 - 2*a15*a7 - 2*a14*a8 - 2*a13*a9 + 23*a23,
 -2*a11*a12 - 2*a10*a13 - 2*a2*a21 - 2*a1*a22 - 2*a0*a23 - 2*a20*a3 - 2*a19*a4 - 2*a1
8*a5 - 2*a17*a6 - 2*a16*a7 - 2*a15*a8 - 2*a14*a9 + 24*a24,
 -a12^2 - 2*a11*a13 - 2*a10*a14 - 2*a2*a22 - 2*a1*a23 - 2*a0*a24 - 2*a21*a3 - 2*a20*a4
- 2*a19*a5 - 2*a18*a6 - 2*a17*a7 - 2*a16*a8 - 2*a15*a9 + 25*a25,
 -2*a12*a13 - 2*a11*a14 - 2*a10*a15 - 2*a2*a23 - 2*a1*a24 - 2*a0*a25 - 2*a22*a3 - 2*a2
1*a4 - 2*a20*a5 - 2*a19*a6 - 2*a18*a7 - 2*a17*a8 - 2*a16*a9 + 26*a26,
 -a13^2 - 2*a12*a14 - 2*a11*a15 - 2*a10*a16 - 2*a2*a24 - 2*a1*a25 - 2*a0*a26 - 2*a23*a
3 - 2*a22*a4 - 2*a21*a5 - 2*a20*a6 - 2*a19*a7 - 2*a18*a8 - 2*a17*a9 + 27*a27,
 -2*a13*a14 - 2*a12*a15 - 2*a11*a16 - 2*a10*a17 - 2*a2*a25 - 2*a1*a26 - 2*a0*a27 - 2*a
24*a3 - 2*a23*a4 - 2*a22*a5 - 2*a21*a6 - 2*a20*a7 - 2*a19*a8 - 2*a18*a9 + 28*a28,
 -a14^2 - 2*a13*a15 - 2*a12*a16 - 2*a11*a17 - 2*a10*a18 - 2*a2*a26 - 2*a1*a27 - 2*a0*a
28 - 2*a25*a3 - 2*a24*a4 - 2*a23*a5 - 2*a22*a6 - 2*a21*a7 - 2*a20*a8 - 2*a19*a9 + 29*a
29,
 -2*a14*a15 - 2*a13*a16 - 2*a12*a17 - 2*a11*a18 - 2*a10*a19 - 2*a2*a27 - 2*a1*a28 - 2*
a0*a29 - 2*a26*a3 - 2*a25*a4 - 2*a24*a5 - 2*a23*a6 - 2*a22*a7 - 2*a21*a8 - 2*a20*a9 +
30*a30,
 -a15^2 - 2*a14*a16 - 2*a13*a17 - 2*a12*a18 - 2*a11*a19 - 2*a10*a20 - 2*a2*a28 - 2*a1*
a29 - 2*a27*a3 - 2*a0*a30 - 2*a26*a4 - 2*a25*a5 - 2*a24*a6 - 2*a23*a7 - 2*a22*a8 - 2*a
21*a9 + 31*a31,
 -2*a15*a16 - 2*a14*a17 - 2*a13*a18 - 2*a12*a19 - 2*a11*a20 - 2*a10*a21 - 2*a2*a29 -
2*a28*a3 - 2*a1*a30 - 2*a0*a31 - 2*a27*a4 - 2*a26*a5 - 2*a25*a6 - 2*a24*a7 - 2*a23*a8
- 2*a22*a9 + 32*a32,
 -a16^2 - 2*a15*a17 - 2*a14*a18 - 2*a13*a19 - 2*a12*a20 - 2*a11*a21 - 2*a10*a22 - 2*a2
9*a3 - 2*a2*a30 - 2*a1*a31 - 2*a0*a32 - 2*a28*a4 - 2*a27*a5 - 2*a26*a6 - 2*a25*a7 - 2*
a24*a8 - 2*a23*a9 + 33*a33,
 -2*a16*a17 - 2*a15*a18 - 2*a14*a19 - 2*a13*a20 - 2*a12*a21 - 2*a11*a22 - 2*a10*a23 -
2*a3*a30 - 2*a2*a31 - 2*a1*a32 - 2*a0*a33 - 2*a29*a4 - 2*a28*a5 - 2*a27*a6 - 2*a26*a7
- 2*a25*a8 - 2*a24*a9 + 34*a34,
 -a17^2 - 2*a16*a18 - 2*a15*a19 - 2*a14*a20 - 2*a13*a21 - 2*a12*a22 - 2*a11*a23 - 2*a1
0*a24 - 2*a3*a31 - 2*a2*a32 - 2*a1*a33 - 2*a0*a34 - 2*a30*a4 - 2*a29*a5 - 2*a28*a6 -
2*a27*a7 - 2*a26*a8 - 2*a25*a9 + 35*a35,
 -2*a17*a18 - 2*a16*a19 - 2*a15*a20 - 2*a14*a21 - 2*a13*a22 - 2*a12*a23 - 2*a11*a24 -
2*a10*a25 - 2*a3*a32 - 2*a2*a33 - 2*a1*a34 - 2*a0*a35 - 2*a31*a4 - 2*a30*a5 - 2*a29*a6
- 2*a28*a7 - 2*a27*a8 - 2*a26*a9 + 36*a36,
 -a18^2 - 2*a17*a19 - 2*a16*a20 - 2*a15*a21 - 2*a14*a22 - 2*a13*a23 - 2*a12*a24 - 2*a1
1*a25 - 2*a10*a26 - 2*a3*a33 - 2*a2*a34 - 2*a1*a35 - 2*a0*a36 - 2*a32*a4 - 2*a31*a5 -
2*a30*a6 - 2*a29*a7 - 2*a28*a8 - 2*a27*a9 + 37*a37,
 -2*a18*a19 - 2*a17*a20 - 2*a16*a21 - 2*a15*a22 - 2*a14*a23 - 2*a13*a24 - 2*a12*a25 -
2*a11*a26 - 2*a10*a27 - 2*a3*a34 - 2*a2*a35 - 2*a1*a36 - 2*a0*a37 - 2*a33*a4 - 2*a32*a
5 - 2*a31*a6 - 2*a30*a7 - 2*a29*a8 - 2*a28*a9 + 38*a38,
 -a19^2 - 2*a18*a20 - 2*a17*a21 - 2*a16*a22 - 2*a15*a23 - 2*a14*a24 - 2*a13*a25 - 2*a1
2*a26 - 2*a11*a27 - 2*a10*a28 - 2*a3*a35 - 2*a2*a36 - 2*a1*a37 - 2*a0*a38 - 2*a34*a4 -
2*a33*a5 - 2*a32*a6 - 2*a31*a7 - 2*a30*a8 - 2*a29*a9 + 39*a39]
```

In [103]:
```
solve(cc+[a0-0],a)
```

Out[103]:
```
[[a0 == 0, a1 == 1, a2 == 0, a3 == (1/3), a4 == 0, a5 == (2/15), a6 == 0, a7 == (17/31
5), a8 == 0, a9 == (62/2835), a10 == 0, a11 == (1382/155925), a12 == 0, a13 == (21844/
6081075), a14 == 0, a15 == (929569/638512875), a16 == 0, a17 == (6404582/10854718875),
a18 == 0, a19 == (443861162/1856156927625), a20 == 0, a21 == (18888466084/194896477400
625), a22 == 0, a23 == (113927491862/2900518163668125), a24 == 0, a25 == (588706684566
04/3698160658676859375), a26 == 0, a27 == (8374643517010684/1298054391195577640625), a
28 == 0, a29 == (689005380505609448/263505041412702261046875), a30 == 0, a31 == (12984
8163681107301953/12252984425690655138679687 5), a32 == 0, a33 == (173664079220990164722
2/40434848604779161957642968 75), a34 == 0, a35 == (418781231495293038913922/2405873491
98436013647975664062 5), a36 == 0, a37 == (56518638202982204522669764/80115587283079192
5447758961328125), a38 == 0, a39 == (32207686319158956594455462/112648292555525012667 3
224649609375)]]
```

Notiamo che ci sono solo i coefficenti dispari. Questi sono 1 1/3 2/15 17/315 62/2835 1382/155925

Possiamo cercare nel database http://oeis.org/ (http://oeis.org/) per un possibile match

In [105]:
```
y=function('y')(x)
desolve(y.diff()==y^2+1,y)
```

Out[105]:
```
arctan(y(x)) == _C + x
```

In [104]:
```

```

Out[104]:

Vogliamo studiare l'equazione lineare $t^2 f''(t) + tf'(t) + (t^2 - 1)f(t) = 0$

In [6]:
```
poleq=derivative(f,t,t)*t^2+t*derivative(f,t)-(1-t^2)*f
```

Out[6]:

In [8]:
```
cc=(poleq.coefficients())
cc
```

Out[8]:
```
[-a0,
 a0 + 3*a2,
 a1 + 8*a3,
 a2 + 15*a4,
 a3 + 24*a5,
 a4 + 35*a6,
 a5 + 48*a7,
 a6 + 63*a8,
 a7 + 80*a9,
 99*a10 + a8,
 120*a11 + a9,
 a10 + 143*a12,
 a11 + 168*a13,
 a12 + 195*a14,
 a13 + 224*a15,
 a14 + 255*a16,
 a15 + 288*a17,
 a16 + 323*a18,
 a17 + 360*a19,
 a18 + 399*a20,
 a19 + 440*a21,
 a20 + 483*a22,
 a21 + 528*a23,
 a22 + 575*a24,
 a23 + 624*a25,
 a24 + 675*a26,
 a25 + 728*a27,
 a26 + 783*a28,
 a27 + 840*a29,
 a28 + 899*a30,
 a29 + 960*a31,
 a30 + 1023*a32,
 a31 + 1088*a33,
 a32 + 1155*a34,
 a33 + 1224*a35,
 a34 + 1295*a36,
 a35 + 1368*a37,
 a36 + 1443*a38,
 a37 + 1520*a39]
```

In [9]:
```
solcoef=solve(cc+[a0-0,a1-1],a)[0]
```

Out[9]:

In [18]:
```
fa=sum(solcoef[j].rhs()*t^j for j in range(40))
```

Out[18]:

In [11]:
```
plot(fa,t,0,5)
```

Out[11]:

In [35]:

```
l=var('l')
solcoef=solve(cc+[a0-0,a1-l],a)[0]
solcoef
```

Out[35]:

```
[a0 == 0,
 a1 == l,
 a2 == 0,
 a3 == -1/6*l + 1/3,
 a4 == 0,
 a5 == 1/120*l - 1/12,
 a6 == 0,
 a7 == -1/5040*l + 1/120,
 a8 == 0,
 a9 == 1/362880*l - 17/36288,
 a10 == 0,
 a11 == -1/39916800*l + 31/1814400,
 a12 == 0,
 a13 == 1/6227020800*l - 1/2280960,
 a14 == 0,
 a15 == -1/1307674368000*l + 5461/653837184000,
 a16 == 0,
 a17 == 1/355687428096000*l - 257/2092278988800,
 a18 == 0,
 a19 == -1/121645100408832000*l + 73/50812489728000,
 a20 == 0,
 a21 == 1/51090942171709440000*l - 1271/92892622130380800,
 a22 == 0,
 a23 == -1/25852016738884976640000*l + 60787/562000363888803840000,
 a24 == 0,
 a25 == 1/15511210043330985984000000*l - 241/334221289449062400000,
 a26 == 0,
 a27 == -1/10888869450418352160768000000*l + 22369621/544443472520917608038400000 0,
 a28 == 0,
 a29 == 1/8841761993739701954543616000000*l - 617093/3048883446117138605015040000 00,
 a30 == 0,
 a31 == -1/8222838654177922817725562880000000*l + 49981/574140389203876750295040000000
0,
 a32 == 0,
 a33 == 1/868331761881188649551819440128000000*l - 16843009/5107833893418756762069526
1184000000,
 a34 == 0,
 a35 == -1/10333147966386144929666651337523200000000*l + 5726623061/516657398319307246
48333256687616000000000,
 a36 == 0,
 a37 == 1/13763753091226345046315979581580902400000000*l - 7957/239054898007776632265 2
782264320000000,
 a38 == 0,
 a39 == -1/20397882081197443358640281739902897356800000000*l + 91625968981/10198941040
5987216793201408699514486784000000000]
```

In [122]:
```
solcoef
```

Out[122]:
```
[a0 == 0,
 a1 == l,
 a2 == 0,
 a3 == -1/8*l,
 a4 == 0,
 a5 == 1/192*l,
 a6 == 0,
 a7 == -1/9216*l,
 a8 == 0,
 a9 == 1/737280*l,
 a10 == 0,
 a11 == -1/88473600*l,
 a12 == 0,
 a13 == 1/14863564800*l,
 a14 == 0,
 a15 == -1/3329438515200*l,
 a16 == 0,
 a17 == 1/958878292377600*l,
 a18 == 0,
 a19 == -1/345196185255936000*l,
 a20 == 0,
 a21 == 1/151886321512611840000*l,
 a22 == 0,
 a23 == -1/80195977758659051520000*l,
 a24 == 0,
 a25 == 1/50042290121403248148480000*l,
 a26 == 0,
 a27 == -1/36430787208381564652093440000*l,
 a28 == 0,
 a29 == 1/30601861255040514307758489600000*l,
 a30 == 0,
 a31 == -1/29377786804838893735448150016000000*l,
 a32 == 0,
 a33 == 1/31963032043664716384167587217408000000*l,
 a34 == 0,
 a35 == -1/39122751221445612854221126754107392000000*l,
 a36 == 0,
 a37 == 1/53519923670937598384574501399618912256000000*l,
 a38 == 0,
 a39 == -1/81350283979825149544553242127420746629120000000*l]
```

In [107]:
```
[x.rhs().subs(l=1/2) for x in solcoef]
```

Out[107]:
```
[0,
 1/2,
 0,
 -1/16,
 0,
 1/384,
 0,
 -1/18432,
 0,
 1/1474560,
 0,
 -1/176947200,
 0,
 1/29727129600,
 0,
 -1/6658877030400,
 0,
 1/1917756584755200,
 0,
 -1/690392370511872000,
 0,
 1/303772643025223680000,
 0,
 -1/160391955517318103040000,
 0,
 1/100084580242806496296960000,
 0,
 -1/72861574416763129304186880000,
 0,
 1/61203722510081028615516979200000,
 0,
 -1/58755573609677787470896300032000000,
 0,
 1/63926064087329432768335174434816000000,
 0,
 -1/78245502442891225708442253508214784000000,
 0,
 1/107039847341875196769149002799237824512000000,
 0,
 -1/162700567959650299089106484254841493258240000000]
```

In [123]:
```
x=var('x')
fb=sum(solcoef[j].rhs().subs(l=1/2)*x^j for j in range(40))
fb
```

Out[123]:
```
-1/162700567959650299089106484254841493258240000000*x^39 + 1/107039847341875196769149002799237824512000000*x^37 - 1/78245502442891225708442253508214784000000*x^35 + 1/63926064087329432768335174434816000000*x^33 - 1/58755573609677787470896300032000000*x^31 + 1/61203722510081028615516979200000*x^29 - 1/72861574416763129304186880000*x^27 + 1/100084580242806496296960000*x^25 - 1/160391955517318103040000*x^23 + 1/303772643025223680000*x^21 - 1/690392370511872000*x^19 + 1/1917756584755200*x^17 - 1/6658877030400*x^15 + 1/29727129600*x^13 - 1/176947200*x^11 + 1/1474560*x^9 - 1/18432*x^7 + 1/384*x^5 - 1/16*x^3 + 1/2*x
```

In [125]:
```
plot([fb,bessel_J(1,x)],x,0,5)
```

Out[125]:

In [13]:
```
```

Out[13]:

Vogliamo ora studiare le equazioni differenziali lineari

In [29]:
```
g(x)=function('g')(x)
```

In [30]:
```
def LL(g):
    return sum(a[j]*diff(g,x,j) for j in range(10))
```

In [33]:
```
LL(g)
```

Out[33]:
```
x |--> a0*g(x) + a1*diff(g(x), x) + a2*diff(g(x), x, x) + a3*diff(g(x), x, x, x) + a4*
diff(g(x), x, x, x, x) + a5*diff(g(x), x, x, x, x, x) + a6*diff(g(x), x, x, x, x, x,
x) + a7*diff(g(x), x, x, x, x, x, x, x) + a8*diff(g(x), x, x, x, x, x, x, x, x) + a9*d
iff(g(x), x, x, x, x, x, x, x, x, x)
```

Osserviamo che:

- L'equazione lineare del primo ordine $f' - \lambda f = 0$ ha come soluzioni le funzioni $ce^{\lambda x}$

- L'equazione lineare del secondo ordine $f'' - \lambda^2 f = 0$ ha come soluzioni le funzioni $c_1 e^{\lambda x}$ e $c_2 e^{-\lambda x}$ e quindi una qualsiasi loro combinazione lineare.

- L'equazione lineare del secondo ordine $f'' + \lambda^2 f = 0$ ha come soluzioni le funzioni $c_1 \sin(\lambda x)$ e $c_2 \cos(\lambda x)$ e quindi una qualsiasi loro combinazione lineare.

In [36]:
```
LL(exp(l*x))
```

Out[36]:
```
a9*l^9*e^(l*x) + a8*l^8*e^(l*x) + a7*l^7*e^(l*x) + a6*l^6*e^(l*x) + a5*l^5*e^(l*x) + a
4*l^4*e^(l*x) + a3*l^3*e^(l*x) + a2*l^2*e^(l*x) + a1*l*e^(l*x) + a0*e^(l*x)
```

In [37]:
```
LL(exp(l*x)).factor()
```

Out[37]:
```
(a9*l^9 + a8*l^8 + a7*l^7 + a6*l^6 + a5*l^5 + a4*l^4 + a3*l^3 + a2*l^2 + a1*l + a0)*e^
(l*x)
```

Pertanto $e^{\lambda x}$ sara' soluzione se e solo se λ e' radice del polinomio
$a9l^9 + a8l^8 + a7l^7 + a6l^6 + a5l^5 + a4l^4 + a3l^3 + a2l^2 + a1l + a0$

In [38]:
```
def LL2(g):
    return diff(g,x,x)-2* diff(g,x)+g
```

In [39]:
```
LL2(g)
```

Out[39]:
```
x |--> g(x) - 2*diff(g(x), x) + diff(g(x), x, x)
```

In [40]:
```
LL2(exp(l*x)).factor()
```

Out[40]:
```
(l - 1)^2*e^(l*x)
```

In [41]:
```
pol=f.derivative(t,t)-2*f.derivative(t)+f
cc=pol.coefficients()
```

Con dato iniziale $f(0) = 1$ e $f'(0) = 1$

In [42]:
```
solve(cc+[a0-1,a1-1],a)
```

Out[42]:
```
[[a0 == 1, a1 == 1, a2 == (1/2), a3 == (1/6), a4 == (1/24), a5 == (1/120), a6 == (1/72
0), a7 == (1/5040), a8 == (1/40320), a9 == (1/362880), a10 == (1/3628800), a11 == (1/3
9916800), a12 == (1/479001600), a13 == (1/6227020800), a14 == (1/87178291200), a15 ==
(1/1307674368000), a16 == (1/20922789888000), a17 == (1/355687428096000), a18 == (1/64
02373705728000), a19 == (1/121645100408832000), a20 == (1/2432902008176640000), a21 ==
(1/51090942171709440000), a22 == (1/1124000727777607680000), a23 == (1/258520167388849
76640000), a24 == (1/620448401733239439360000), a25 == (1/15511210043330985984000000),
a26 == (1/403291461126605635584000000), a27 == (1/10888869450418352160768000000), a28
== (1/304888344611713860501504000000), a29 == (1/8841761993739701954543616000000), a30
== (1/265252859812191058636308480000000), a31 == (1/822283865417792281772556288000000
0), a32 == (1/263130836933693530167218012160000000), a33 == (1/868331761881188649551819
4401280000000), a34 == (1/295232799039604140847618609643520000000), a35 == (1/1033314
7966386144929666651337523200000000), a36 == (1/371993326789901217467999448150835200000
000), a37 == (1/13763753091226345046315979581580902400000000), a38 == (1/523022617466
01111760007224100074291200000000), a39 == (1/20397882081197443358640281739902897356800
000000)]]
```

Che e' proprio la soluzione e^x

Scegliamo ora come dato iniziale $f(0) = 0$ e $f'(0) = 1$

In [43]:
```
solve(cc+[a0-0,a1-1],a)
```

Out[43]:
```
[[a0 == 0, a1 == 1, a2 == 1, a3 == (1/2), a4 == (1/6), a5 == (1/24), a6 == (1/120), a7
== (1/720), a8 == (1/5040), a9 == (1/40320), a10 == (1/362880), a11 == (1/3628800), a1
2 == (1/39916800), a13 == (1/479001600), a14 == (1/6227020800), a15 == (1/8717829120
0), a16 == (1/1307674368000), a17 == (1/20922789888000), a18 == (1/355687428096000), a
19 == (1/6402373705728000), a20 == (1/121645100408832000), a21 == (1/24329020081766400
00), a22 == (1/51090942171709440000), a23 == (1/1124000727777607680000), a24 == (1/258
52016738884976640000), a25 == (1/620448401733239439360000), a26 == (1/1551121004333098
5984000000), a27 == (1/403291461126605635584000000), a28 == (1/10888869450418352160768
000000), a29 == (1/304888344611713860501504000000), a30 == (1/884176199373970195454361
6000000), a31 == (1/265252859812191058636308480000000), a32 == (1/822283865417792281772
5562880000000), a33 == (1/263130836933693530167218012160000000), a34 == (1/8683317618
811886495518194401280000000), a35 == (1/295232799039604140847618609643520000000), a36
== (1/10333147966386144929666651337523200000000), a37 == (1/371993326789901217467999448
1508352000000000), a38 == (1/13763753091226345046315979581580902400000000), a39 == (1/
523022617466601111760007224100074291200000000)]]
```

In [44]:
```
LL2(x*exp(x))
```

Out[44]:
```
0
```

In [ ]:
```

```

In [ ]:
```

```